Wilson loop and dimensional reduction in noncommutative gauge theories

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(Received 2 May 2001; published 25 September 2001)

Using the anti–de Sitter (AdS) conformal field theory correspondence we study the UV behavior of Wilson loops in various noncommutative gauge theories. We get an area law in most cases and try to identify its origin. In the D3 case, we may identify the the origin as the D1 dominance over the D3: as we go to the boundary of AdS space, the effect of the flux of the D3 charge is highly suppressed, while the flux due to the D1 charge is enhanced. So near the boundary the theory is more like a theory on a D1-brane than that on a D3-brane. This phenomena is closely related to dimensional reduction due to the strong magnetic field in the charged particle in the magnetic field. The linear potential is not due to the confinement by IR effect but is the analogue of Coulomb's potential in $1+1$ dimensions.

DOI: 10.1103/PhysRevD.64.086002 PACS number(s): 11.25.−w, 11.15.Pg

I. INTRODUCTION

Recently, Maldacena [1] (see Refs. [2,3] for a review) conjectured that string theory on anti–de Sitter (AdS) spacetime is dual to SU(N) super Yang-Mills (SYM) theory, named AdS conformal field theory (CFT) correspondence. If we turn on a Neveu-Schwarz (NS) B field on the N folding D-brane world volume, the low energy effective theory is equivalent to a noncommutative U(N) super Yang-Mills (NCSYM) theory [4–10]. The corresponding dual gravity solution with a nonvanishing B field was constructed in Ref. [11] as a bound state of branes.

A Wilson loop can be calculated by the minimal area whose boundary is the given Wilson loop [12]. In the nonvanishing B-field background, it is observed that the string tilts from its usual direction (orthogonal to the boundary of AdS) by a certain angle so that the length of the string along the boundary is infinite [13]. Wilson loop that goes deep into the near horizon (IR) was found to give a Coulombic potential. In Ref. [14], it was observed that for a string moving with special velocity, the tilting angle is zero and the effect of the noncommutativity is merely renormalizing the Coulomb potential. So far, however, it is not clear why one should calculate the Wilson loop behavior at a fine tuned velocity. The unusual feature of the Wilson loops in the presence of the B field are associated with nonlocality of the boundary gauge theory and the lack of the gauge invariance of the Wilson loop [15–17]. The super gravity solution is not asymptotically AdS_5 space: the noncommutative directions shrink near the boundary. So there are some skepticism whether one can extract any physics out of the Wilson loop in noncommutative gauge theory.

In the recent paper of Dhar and Kitazawa [18], it is noticed that if we place the boundary at the finite position $u=\Lambda$, we can find a branch that gives the Coulomb potential and the situation looks as a small deformation of the commutative case. The price for having such branch is that the string configuration is not uniquely determined for a given length of Wilson line unless one put the probe brane at the noncommutative scale $u\sim 1/a$. If we put the probe brane at $1/a$, we are cutting out all the ''noncommutative region'' (strong B-field region) in the bulk. Therefore it is not surprising that they get the Coulomb's law for large Wilson line. Small Wilson line, whose Nambu-Goto string stays near the boundary, can ''feel'' the effect of the B field. In this case they find the area law. So they got the transition from Coulomb to area law as the size of the wilson line changes from large to small. Although interesting, the physics of the area law is not clear at all in this approach, especially because they cut out the all the strong B-field region.

In this paper, we try to identify the mechanism of the area law. If the area law is a character of the the noncommutativity, we can expect that we should get it for any Wilson line which stay in the large B field region. So we do not put the boundary at the finite u. We put it at infinity as usual. As a consequence, the Coulomb branch, is not available to us. We will probe the noncommutative regime where the minimum point of the string u_0 is larger than the noncommutatve scale $1/a$ so that entire Nambu-Goto string of the Wilson line is in the strong B-field region. We will find that the Wilson line follows universal area law. This is contrasted with the case of commutative case, where temporal loop gives Coulomb's law while spatial loop gives an area law [1,3,19–21]. In the presence B field case, we will show that we get area law for both case.

In the D3 case, we may identify the the origin as the D1 dominance [22,23] over the D3: as we go to the boundary of the AdS space, the effect of the flux of the D3 charge is highly suppressed, while the flux due to the D1 charge is enhanced. So near the boundary, the theory is more similar to a theory on a D1-brane than that on a D3-brane. This phenomena is closely related to the dimensional reduction due to the strong magnetic field in the charged particle in the magnetic field. Then, the linear potential is not due to the confinement by IR effect but is the ''analogue'' of Coulomb's potential in $1+1$ dimensions.

This paper is organized as follows. In Sec. II, we review the gravity dual of noncommutative gauge theory and its scaling symmetries. In Sec. III, we calculate the Wilson loop in the UV regime for various cases including finite tempera-

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tures, spatial as well as temporal loops, D-instanton background and other D_p-brane cases. We get an area law almost universally if time is not noncommutative. In Sec. IV, we give a physical interpretation for the area law of $(3+1)$-dimensional noncommutative gauge theories as D1 dominance and dimensional reduction due to the magnetic field. We summarize and conclude in Sec. V.

II. GRAVITY DUAL OF THE NONCOMMUTATIVE GAUGE THEORY AND ITS SCALING SYMMETRY

Let us first consider the zero temperature case of a D3-brane with constant B field parallel to the brane. Its low energy effective world-volume theory is described by noncommutative Yang-Mills theory. The gravity dual solution in string frame is given in Refs. [11,13,24]. Its solution is bound state solution of D3 and D1-branes and is given by

$$ds_s^2=f^{-1/2}[-dx_0^2+dx_1^2+h(dx_2^2+dx_3^3)]+f^{1/2}(dr^2+r^2d\Omega_5^2),$$

$$f=1+\frac{\alpha'^2R^4}{r^4},\quad h^{-1}=\sin^2\theta f^{-1}+\cos^2\theta,$$

$$B_{23}=\frac{\sin\theta}{\cos\theta}f^{-1}h,$$

$$e^{2\phi}=g^2h,$$

$$F_{01r}=\frac{1}{g}\sin\theta\partial_r f^{-1},\quad F_{0123r}=\frac{1}{g}\cos\theta h\partial_r f^{-1}. \tag{2.1}$$

The above solution is asymptotically flat for $r\to\infty$ and they have a horizon at $r=0$. In region near $r=0$ the solution has a form $AdS_5\times S^5$. In order to obtain noncommutative field theory we should take the B field to infinity. In the decoupling limit $\alpha'\to 0$ with finite fixed variables

$$u=\frac{r}{\alpha'R^2},\quad \tilde{b}=\alpha'\tan\theta,\quad \tilde{x}_{2,3}=\frac{\tilde{b}}{\alpha'}x_{2,3},\quad \hat{g}=\frac{\tilde{b}}{\alpha'}g, \tag{2.2}$$

and the metric becomes

$$ds^2=\alpha'R^2\left[u^2\{-dx_0^2+dx_1^2+\hat{h}(d\tilde{x}_2^2+d\tilde{x}_3^2)\}+\frac{du^2}{u^2}+d\Omega_5^2\right], \tag{2.3}$$

$$\hat{h}=\frac{1}{a+a^4u^4},\quad a^2=\tilde{b}R^2,$$

$$\tilde{B}_{23}=B_\infty\frac{a^4u^4}{1+a^4u^4},\quad B_\infty=\frac{\alpha'}{\tilde{b}}=\alpha'\frac{R^2}{a^2},$$

$$e^{2\phi}=\hat{g}^2\hat{h},$$

$$A_{01}=\alpha'\frac{\tilde{b}}{\hat{g}}u^4R^4,$$

$$\tilde{F}_{0123u}=\alpha'^2\frac{\hat{h}}{\hat{g}}\partial_u(u^4R^4),$$

where $\hat{g}$ is the value of the string coupling in the IR and $R^4=4\pi\hat{g}N$. This is the gravity dual solution to NCSYM. For small u which corresponds to IR regime of gauge theory the metric reduces to ordinary $AdS_5\times S^5$. This is consistent with the expectation that the noncommutative Yang-Mills theory reduces to ordinary Yang-Mills theory at long distances (IR). The solution starts deviating from the AdS space at $u\sim 1/a$, i.e., at a distance scale $\sim R\sqrt{\tilde{b}}$. For large R^4 where supergravity limit is valid, this is greater than the naively expected distance scale of $L\sim\sqrt{\tilde{b}}$. So the effects of noncommutativity is visible at longer distances than naively expected. The metric has a boundary at $u=\infty$. As we approach to the boundary, the physical scale of the 2,3 direction shrinks. In this region it seems that only the D1-brane is relevent. The noncommutative nature arise from the fact that the position of D1 in D3 is not fixed but widely fluctuating.

We now discuss the symmetry property of the metric. In the absence of B field, the metric is that of the well known $AdS_5\times S^5$:

$$ds^2=\alpha'R^2\left[u^2(-dx_0^2+dx_1^2+dx_2^2+dx_3^2+dx_3^2)+\frac{du^2}{u^2}+d\Omega_5^2\right]. \tag{2.4}$$

This metric has a rescaling symmetry at the boundary

$$x^\mu\to\lambda x^\mu\ \ (\mu=0,1,2,3)\quad\text{and}\quad u\to\frac{u}{\lambda}. \tag{2.5}$$

This symmetry is associated with the UV/IR: large in x corresponds to the small in u. In the presence of B field, however, the metric near the boundary has the form

$$ds^2=\alpha'R^2\left[u^2(-dx_0^2+dx_1^2)+\frac{1}{u^2}(dx_2^2+dx_3^2)+\frac{du^2}{u^2}+d\Omega_5^2\right]. \tag{2.6}$$

At the boundary the noncommutative directions shrink and the metric effectively becomes that of AdS_3. This has the scaling symmetry at the boundary

$$x^{0,1}\to\frac{1}{\lambda}x^{0,1},\quad x^{2,3}\to\lambda x^{2,3},\quad\text{and}\quad u\to\lambda u, \tag{2.7}$$

which is slightly different from that of the zero B field case. Therefore the coordinate distance L along the noncommutative direction near the boundary $u=U$ corresponds to the physical length $x^{2,3}/aU$ [18].

III. WILSON LOOPS IN VARIOUS CASES

In this section we consider temporal Wilson loop at finite temperature in noncommutative gauge theory. The gravity dual is a nonextremal blackhole background with B dependence [13]. The metric is given by

$$ds^2=\alpha' R^2\left[u^2\left\{-\left(1-\frac{u_h^4}{u^4}\right)dx_0^2+dx_1^2+\hat{h}(dx_2^2+dx_3^2)\right\}+\frac{du^2}{u^2(1-u_h^4/u^4)}+d\Omega_5^2\right]. \tag{3.1}$$

Here tildes are omitted for convenience. String theory on this background should provide a dual description of noncommutative Yang-Mill theory at finite temperature. For small u, the metric is reduced to that of the AdS-Schwarzschild black hole. Let u_0 be the smallest possible value of u on the Wilson loop in the bulk. This gravity dual solution can be trusted when the following conditions are satisfied: Small string coupling

$$e^{\phi}=\frac{\hat{g}}{\sqrt{1+a^4u_0^4}}\ll 1, \tag{3.2}$$

small curvature

$$\hat{g}_{YM}^2N=\hat{g}N\gg 1. \tag{3.3}$$

We know from the above gravity solution, that $au_0\sim 1$ is a transition region from AdS_5 black hole region to dimensionally reduced AdS_3 region. Let u_0 be the minimal value available to the string configuration. The noncommutative effect is relevant to $au_0\gg 1$ (UV) region. Since it is expected to get Coulomb's law for $au_0\ll 1$ (IR) where the noncommutaive effect is invisible, it is expected to get something else for the quark-antiquark potential, similar to an area law. We will show indeed this is so by calculating the Wilson loops in various cases and also we will find necessary condition for this to happen. There is agreement that IR behavior is Coulombic [13,14,18], while there are different opinions for the UV behavior. Therefore our interest will be for $au_0\gg 1$ (UV) where the noncommutative effect is manifest in the metric behavior ($a\sim\tilde{b}$).

A. Wilson loops in noncommuative gauge theory at finite temperature

Temporal loop. Now consider classical string world-sheet action, Nambu-Goto action, which is given by

$$S=\frac{1}{2\pi\alpha'}\int d\sigma d\tau\sqrt{\det(h_{\alpha\beta})}, \tag{3.4}$$

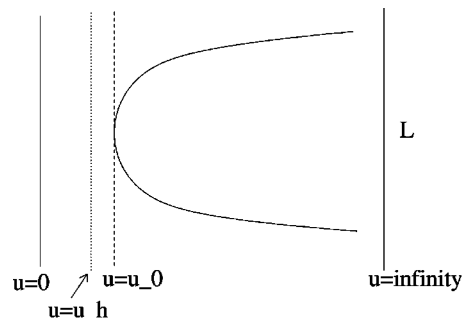

FIG. 1. $Q\bar{Q}$ separation L grows indefinitely since $x_3=ku$ as we approaches to the boundary.

where $h_{\alpha\beta}=G_{MN}\partial_\alpha X^M\partial_\beta X^N$. We choose static gauge as $\tau=x_0$, $\sigma=x_2$, $u=u(\sigma)$. Then on the background of Eq. (3.1), we have

$$h_{\tau\tau}=\alpha' R^2u^2\left(1-\frac{u_h^4}{u^4}\right),$$

$$h_{\tau\sigma}=h_{\sigma\tau}=0,$$

$$h_{\sigma\sigma}=\alpha' R^2u^2\hat{h}+\alpha' u^{-2}\left(1-\frac{u_h^4}{u^4}\right)^{-1}(\partial_\sigma u)^2. \tag{3.5}$$

So the Nambu-Goto action can be written as

$$S=\frac{R^2}{2\pi}T\int dx_2\sqrt{\hat{h}(u^4-u_h^4)+(\partial_{x_2}u)^2}. \tag{3.6}$$

The action does not explicitly depend on x_2 so it gives

$$\frac{\hat{h}(u^4-u_h^4)}{\sqrt{\hat{h}(u^4-u_h^4)+(\partial_{x_2}u)^2}}=c. \tag{3.7}$$

At $u=u_0$, where it is the closest point to the horizon, $\partial_{x_2}u=0$ and $\hat{h}\to\hat{h}_0$. Then we can determine c

$$c^2=\hat{h}_0(u_0^4-u_h^4). \tag{3.8}$$

This allows us to write x_2 as a function of u

$$x_2=\sqrt{\frac{u_0^4-u_h^4}{1+a^4u_h^4}}\int du\frac{1+a^4u^4}{\sqrt{(u^4-u_0^4)(u^4-u_h^4)}}. \tag{3.9}$$

For large u,

$$x_2\sim a^4\sqrt{\frac{u_0^4-u_h^4}{1+a^4u_h^4}}u=ku, \tag{3.10}$$

where k can be interpreted as a slope for large u. This implies that we cannot fix the position of the string at infinity since x_2 grows linearly with u (see Fig. 1). This dependence of u is associated to nonlocality [25] of noncommutative theory. In the IR region we get the Coulombic potential by similar calculation of Ref. [14]. However, we will soon see that in the UV region $(au_0 \gg 1)$ we get different one.

L and u_0 are related by

$$\frac{L}{2} = x_2(u \to \infty)$$
$$= \sqrt{\frac{1-\frac{u_h^4}{u_0^4}}{1+a^4u_h^4}}\frac{1}{u_0}\int_1^\infty dy \frac{1+a^4u_0^4y^4}{\sqrt{(y^4-1)\left(y^4-\frac{u_h^4}{u_0^4}\right)}}, \tag{3.11}$$

where $y = u/u_0$. The energy of this configuration is given by

$$E = \frac{R^2}{\pi}\sqrt{\frac{1+a^4u_0^4}{1+a^4u_h^4}}u_0\int_1^\infty dy\sqrt{\frac{y^4-u_h^4/u_0^4}{y^4-1}}. \tag{3.12}$$

For $u_h a \leqslant u_0 a \ll 1$, it is known that we get the screened Coulomb potential [19,12].

We are currently interested in the UV regime $(au_0 \gg 1)$ where the noncommutative effect is crucial. When we consider $au_0 \gg 1$ case (equivalently $u_h/u_0 \to 0$), L and E are, respectively, given by

$$\frac{L}{2} \sim \frac{1}{u_0}\left[a^4u_0^4\int_1^\infty dy\frac{y^2}{\sqrt{y^4-1}}\right] \quad \text{and}$$

$$E \sim \frac{R^2}{\pi}a^2u_0^2 \cdot u_0\int_1^\infty dy\frac{y^2}{\sqrt{y^4-1}}. \tag{3.13}$$

Then the relation between E and L can be written as

$$E = \frac{R^2}{\pi a^2}L. \tag{3.14}$$

Both separation length and energy are infinite and there is no canonical way to renormalize the separation length. If all what we want is the E-L relation, the only reasonable way to regularize both quantity is to put a cutoff in u. The question is whether we can give a physical interpretation to this. We will try this in Sec. IV. For the moment, we just calculate the Wilson loops for various other situations.

B. Spatial loop

Spatial Wilson loops in high temperature are interesting since they can be considered as ordinary Wilson loops of Euclidean field theory in one less dimension.

1. In three dimensions

Let us start with the Euclidean nonextremal D3-brane metric with B_{23} fields. The metric has the same form as before

$$ds^2 = \alpha' R^2\left[u^2\left\{\left(1-\frac{u_h^4}{u^4}\right)dt_E^2 + dx_1^2 + \hat{h}(dx_2^2+dx_3^2)\right\} + \frac{du^2}{u^2(1-u_h^4/u^4)} + d\Omega_5^2\right]. \tag{3.15}$$

Compactify this four-dimensional theory on $\mathbf{S}^1$ of t_E by periodically identify its period with the inverse Hawking temperature proportional to u_h and take the high-temperature limit so that the radius becomes zero. The resulting effective theory is then interpreted as Euclideanized three-dimensional noncommutative field theory. The circle compactification breaks both supersymmetry and conformal symmetry. After compactifying t_E, the resulting three-dimensional theory is described by coordinates x_1, x_2, and x_3. Among them, for instance, x_1 can be considered as a Euclideanized time and x_2, x_3 as spatial coordinates. For the spatial loop, the string configuration we use is $\tau = x_1$, $\sigma = x_2$, and $u = u(\sigma)$. Then the Nambu-Goto action becomes

$$S = \frac{TR^2}{2\pi}\int dx_2\sqrt{u^4\hat{h} + (1-u_h^4/u^4)^{-1}(\partial_{x_2}u)^2}, \tag{3.16}$$

where $\hat{h} = 1/(1+a^4u^4)$. Then it is easy to show that the separation length

$$L = \frac{2}{u_0}\int_1^\infty dy\frac{1+a^4u_0^4y^4}{\sqrt{(y^4-1)(y^4-u_h^4/u_0^4)}}. \tag{3.17}$$

and energy

$$E = \frac{R^2u_0\sqrt{1+a^4u_0^4}}{2\pi}\int_1^\infty dy\frac{y^4}{\sqrt{(y^4-1)(y^4-u_h^4/u_0^4)}}. \tag{3.18}$$

From these, the relation between E and L when the noncommutative parameter au_0 is

$$E \sim \frac{R^2}{2\pi a^2}L. \tag{3.19}$$

In the case of $B=0$, the spatial loop is gives an area law, while the temporal loops are not. Here $(B \neq 0)$, both give the area law when the noncommutative parameter is large.

2. Four-dimensional case

Here we start with nonextremal D4-brane. The metric with B fields [13] is given by

$$ds^2 = f^{-1/2}[dx_0^2 + dx_1^2 + h(dx_2^2+dx_3^2) + dx_4^2] + f^{1/2}(dr^2 + r^2d\Omega_4^2), \tag{3.20}$$

where

$$f=1+\frac{\alpha'^{3/2}R^3}{r^3},\quad h^{-1}=f^{-1}\sin^2\theta+\cos^2\theta,$$

$$B_{23}=f^{-1}h\tan\theta,$$

$$e^{2\phi}=g^2f^{-1/2}h. \tag{3.21}$$

The decoupling limit exists and is given by

$$r=\alpha' R^2u,\quad \alpha'\to 0,$$

$$\tan\theta=\frac{\tilde b}{\alpha'},\quad x_i=\frac{\alpha'}{\tilde b}\tilde x_i,\quad a^3=\tilde b^2\alpha'^{-1/2}R^3,$$

$$\tilde B_{23}=\frac{\alpha'}{\tilde b}\frac{a^3u^3}{1+a^3u^3}. \tag{3.22}$$

For the nonextremal case the metric is simply written as

$$ds^2=\alpha' R^2\Bigg[(\alpha'^{1/4}R^{1/2}u^{1/2})^{-1}u^2\Bigg\{\Bigg(1-\frac{u_h^3}{u^3}\Bigg)dt^2+dx_1^2+\hat h(dx_2^2+dx_3^2)+dx_4^2\Bigg\}+(\alpha'^{1/4}R^{1/2}u^{1/2})\times\Bigg(\frac{du^2}{u^2(1-u_h^3/u^3)}+d\Omega_4^2\Bigg)\Bigg], \tag{3.23}$$

where $\hat h=1/(1+a^3u^3)$ and tildes are ommited for convenience. For static string configuration $\tau=x_1$, $\sigma=x_2$, $u=u(\sigma)$, the Nambu-Goto action can be written as

$$S=\frac{TR^2}{2\pi}\int dx_2\sqrt{\alpha'^{-1/2}R^{-1}u^3\hat h+(1-u_h^3/u^3)^{-1}(\partial_{x_2}u)^2}. \tag{3.24}$$

The first integral is

$$\frac{\alpha'^{1/2}R^{-1}u^3\hat h}{\sqrt{\alpha'^{-1/2}R^{-1}u^3\hat h+(1-u_h^3/u^3)^{-1}(\partial_{x_2}u)^2}}=c, \tag{3.25}$$

where $c^2=\alpha'^{-1/2}R^{-1}u_0^3\hat h_0$. After some calculation, the separation length

$$L=\frac{2u_0^{-1/2}}{\sqrt{\alpha'^{-1/2}R^{-1}}}\int_1^\infty dy\,\frac{1+a^3u_0^3y^3}{\sqrt{(y^3-1)(y^3-u_h^3/u_0^3)}} \tag{3.26}$$

and the energy

$$E=\frac{R^2u_0\sqrt{1+a^3u_0^3}}{\pi}\int_1^\infty dy\,\frac{y^3}{\sqrt{(y^3-1)(y^3-u_h^3/u_0^3)}}. \tag{3.27}$$

For the UV regime $au_0\gg 1$, we again get an area law:

$$E=\left(\frac{R^3}{\alpha'^{1/2}\pi^2a^3}\right)^{1/2}L. \tag{3.28}$$

C. Wilson loop in the *D*-instanton background

In this section we consider one more example: quark-antiquark potential in D-instanton background with constant B field. The gravity dual solution for $B=0$ was considered in Ref. [26]. It is easy to turn on B field for this background. First rotate the D-instanton background and then T dualize it. Then the resulting metric [27] is given by

$$ds^2=H^{1/2}[f^{-1/2}\{dt^2+dx_1^2+h(dx_2^2+dx_3^2)\}+f^{1/2}(dr^2+r^2d\Omega_5^2)], \tag{3.29}$$

where

$$f=1+\frac{\alpha'^2}{r^4},\quad H=1+\frac{q\alpha'^4R^4}{r^4},$$

$$e^{2\phi}=g^2hH,\quad B_{23}=f^{-1}hH\tan\theta,$$

$$h=\frac{1}{Hf^{-1}\sin^2\theta+\cos^3\theta}. \tag{3.30}$$

This D-instanton is smeared over the D3-brane world volume. This solution is T dual to D4+D0 or D5+D1 (with B field) brane configuration. In the decoupling limit

$$r=\alpha' R^2u,\quad \alpha'\to 0,$$

$$\tan\theta=\frac{\tilde b}{\alpha'},\quad f\to(\alpha'^2R^4u^4)^{-1},$$

$$H\to 1+\frac{q}{R^4u^4},\quad a^2=\tilde bR^2,$$

$$h\to\frac{\tilde b^2}{\alpha'^2(1+Ha^4u^4)},$$

$$B_{23}\to H\frac{a^4u^4}{(1+Ha^4u^4)},$$

$$x_{0,1}=\tilde x_{0,1},\quad x_{2,3}=\frac{\alpha'}{\tilde b}\tilde x_{2,3}. \tag{3.31}$$

So the metric becomes

$$ds^2=\alpha' R^2\left(1+\frac{q}{R^4u^4}\right)^{1/2}\Bigg[u^2\{d\tilde x_0^2+d\tilde x_1^2+\hat h(d\tilde x_2^2+d\tilde x_3^2)\}+\left(\frac{du^2}{u^2}+d\Omega_5^2\right)\Bigg], \tag{3.32}$$

where $\hat h=1/(1+Ha^4u^4)$ and $H=1+q/R^4u^4$. The metric becomes flat $\mathbf{R}^{10}$ as $u\to 0$ while it deviates from flat as u become large. When B field is zero, this metric solution describes wormhole solution which connects flat space ($\mathbf{R}^{10}$) in $u\to 0$ with $\mathrm{AdS}^5\times S_5$ in $u\to\infty$.

For the static string configuration, $\tau=x_0$, $\sigma=x_2$, and $u=u(\sigma)$, the Nambu-Goto action can be written as

$$S=\frac{R^2T}{2\pi}\int dx_2\sqrt{H\{u^4\hat{h}+(\partial_{x_2}u)^2\}}, \tag{3.33}$$

which gives the following first integral:

$$\frac{\sqrt{H}u^4\hat{h}}{\sqrt{u^4\hat{h}+(\partial_{x_2}u)^2}}=c, \tag{3.34}$$

where $c^2=H_0u_0^4\hat{h}_0$. L and E are given by

$$\frac{L}{2}=\frac{\sqrt{(1+q/R^4u_0^4)}}{u_0}\int_1^\infty dy\frac{(1+a^4y^4u_0^4+qa^4/R^4)}{y^2\sqrt{y^4-1}},$$

$$E=\frac{R^2}{\pi}\sqrt{1+a^4u_0^4+qa^4/R^4}u_0\int_1^\infty dy\frac{(y^2+q/R^4y^2u_0^4)}{\sqrt{y^4-1}}. \tag{3.35}$$

Here our interest lies in the region where $au_0\gg 1$ and $u_0\gg u_h$,

$$L\sim a^4u_0^3\int_1^\infty dy\frac{y^2}{\sqrt{y^4-1}},\quad \text{and}\quad E\sim\frac{a^2R^2u_0^3}{\pi}\int_1^\infty dy\frac{y^2}{\sqrt{y^4-1}}, \tag{3.36}$$

which after elimination of u_0 gives

$$E=\frac{R^2}{\pi a^2}L. \tag{3.37}$$

One should notice that in UV region, $u>u_0>1/a$, the D-instanton effect is negligible and the noncommutativity effect is dominant. The fact that the D-instanton charge is proportional to α'^4 is crucial to neglect the D-instanton effect in the UV region.

For $B=0$, it is known [26] that the large L correspond to $(u_0\to 0)$ and it leads to an area law. To compare this and above case let us look at some details. In this case L and E is given by

$$\frac{L}{2}=\sqrt{1+\frac{q}{Ru_0^4}}\frac{1}{u_0}\int_1^\infty dy\frac{1}{y^2\sqrt{y^4-1}}=\frac{2\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}\sqrt{1+\frac{q}{R^4u_0^4}}\frac{1}{u_0},$$

$$\begin{aligned}E&=\frac{R^2}{\pi}\left[\int_1^\infty dy\left(\frac{y^2}{\sqrt{y^4-1}}-1\right)-1\right]+\frac{R^2}{\pi}\frac{1}{u_0^3}\int_1^\infty dy\frac{1}{y^2\sqrt{y^4-1}}\\&=-\frac{2R^2u_0\sqrt{\pi}}{\Gamma(\frac{1}{4})^2}+\frac{\sqrt{2}R^2\sqrt{\pi}}{\Gamma(q/4)^2u_0^3}.\end{aligned} \tag{3.38}$$

As $u_0\to 0$, $L\to\infty$, we have

$$E=\frac{R^4}{2\pi\sqrt{q}}L. \tag{3.39}$$

Although Eqs. (3.37) and (3.39) gives similar results the details are very different. $B=0$ case is the IR result ($u_0\ll 1$) and is a consequence of dynamics, while $B\neq 0$ case is the UV result and kinematical.

General D_p brane cases

In order to study more general case, we look at the Wilson loop in higher dimensional noncommutative Yang-Mill theories. The dual gravity metric is given by [13]

$$ds^2=H_p^{-1/2}[h_0(-dx_0^2+dx_1^2)+h_1(dx_2^2+dx_3^2)+\cdots]+H_p^{1/2}[du^2+u^2d\Omega_5^2] \tag{3.40}$$

with $H_p\sim 1/u^{7-p}$ and $h_i=1/(a_iu)^4$. We again, do not need to consider the black holes since we are interested in large $au_0\gg 1$ case. The general string action is

$$S\sim T\int dx\sqrt{h_0[H^{-1}h+(\partial_xu)^2]}. \tag{3.41}$$

Here we assumed that the string is along x direction which is one of the noncommutative planes. We immediately see that as $u\to\infty$, $H^{-1}h\to$const, resulting in the linear potential if and only if $h_0=$const, namely, of no B_{01} field is applied. This can be easily verified if we observe that when h_0 is constant the action (3.41) leads to the first integral

$$(\partial_xu)^2+f(1-f/f_0)=0, \tag{3.42}$$

where $f(u)=H^{-1}(u)h(u)$ and f_0 is the value of f at $u=u_0$. The qualitative behavior of the solution can be read off from the particle moving with zero energy under the potential $f(1-f/f_0)$, see Fig. 2. The effect of D3-brane charge [F_5 flux $H(u)$] is to pull out the particle to the boundary of AdS,

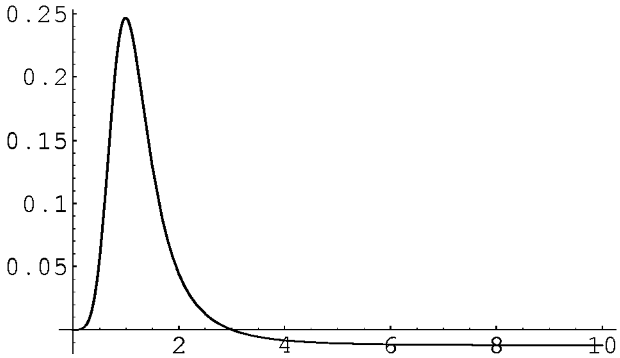

FIG. 2. The minimal surface problem is reduced to the motion of a classical particle moving with 0 total energy in a potential $V=f(1-f/f_0)$. The potential peak is near the $1/a$. In the plot, we set $a=1,\ u_0=3$.

while that of NS-NS charge [B_{23} field $h(u)$] is to pull in the particle into the horizon. In terms of boundary variable, the former expands the x^μ along the brane directions, while the latter shrink the x^2,x^3 plane. The essence of the phenomena is the exact cancellation of two effect in the asymptotic region. Since both H and h are based on the harmonic power $u^{-(7-p)}$ of the transverse dimensions, this is unavoidable in the region where those terms are dominant. The net effect is such that the particle has a constant speed, or the Wilson loop has a constant slope.

IV. D1 DOMINANCE AND DIMENSIONAL REDUCTION

Now we want to figure out the origin of the linear behavior of the potential in more physical terms for the interested D3 case. Consider the metric for the following class:

$$ds^2=\alpha' R^2\left[u^2h_0(-dx_0^2+dx_1^2)+u^2h_1(dx_2^2+dx_3^2+dx_3^2)+\frac{du^2}{u^2}+d\Omega_5^2\right], \tag{4.1}$$

where $h_i=1/[1+(a_iu)^4]$. For large $au_0\gg 1$ and $u_0\gg u_h$, we do not need to care about the black hole effect. The noncommutativity effect (B field effect) is dominant. The string action is

$$S\sim T\int dx_2\sqrt{h_0[u^4h_1+(\partial_x u)^2]}. \tag{4.2}$$

For the case we considered, $h_0=1$ and $h_1\sim 1/u^4$, so that the action becomes

$$S\sim T/a\int \sqrt{1+(du/dx)^2}, \tag{4.3}$$

after the scaling $u\to u/a$ and $x\to ax$. Therefore the energy is proportional to the line element of flat space. The shortest length is for the straight line. Therefore since the line is not orthogonal to the boundary, the energy has to be proportional to the linear length in the x direction. What happens if we turn on B_{01} also? In this case nontrivial h_0 arise [13]. The metric for large u region is that of $AdS_5\times S^5$ [13]:

$$ds^2=\alpha' R^2\left[\frac{-dx_0^2+dx_1^2+dx_2^2+dx_3^2+du^2}{u^2}+d\Omega_5^2\right] \tag{4.4}$$

which is the metric of $AdS_5\times S^5$ (but in Poincare coordinate). So it is similar to the near horizon geometry of distributed D-instanton over the D3 [13,26,28]. The boundary $u=\infty$ is now at the AdS horizon. In fact one can show that the critical path with the given boundary condition does not exist. From the calculational point of view, what is crucial for the area law is the absence of h factor in the g_{tt}. However this means that g_{11} is also free of the h factor. Therefore we have to have one spatial direction along which B field is not applied to get the area law. According to the supergravity solution (2.1), there is a D1 branes along x^1 direction. Furthermore, near the boundary, the existence of the D3-brane is suppressed by $h\sim 1/u^4$ factor relative to the D1-branes:

$$F_{01r}=\frac{1}{g}\sin\theta\partial_r f^{-1},\quad F_{0123r}=\frac{1}{g}\cos\theta h\partial_r f^{-1}. \tag{4.5}$$

In near horizon limit,

$$F_{01u}=\alpha'\frac{\tilde{b}}{\hat{g}}\partial_u(u^4R^4),$$

$$\tilde{F}_{0123u}=\hat{h}\cdot\frac{\alpha'^2}{\hat{g}}\partial_u(u^4R^4). \tag{4.6}$$

Another manifestation of the D1 dominance [22,23] near the boundary is the metric itself. The near horizon limit of the metric shows that g_{22},g_{33} is suppressed by the same factor $h\sim 1/u^4$ compared to the g_{11}. In fact this suppression of the noncommutative direction is the motivation to begin this work. The noncommutativity in x^2,x^3 can be interpreted as the fluctuation of the the location of the D1-brane along those directions. In fact this is origin of the fluctuation of the end point of the Wilson line noted in Ref. [13].

One may further understand the behavior of Wilson loop by considering the open string as dipole [29,30] and taking the analogy to the charged particle in magnetic field. In case of charged particle, when F_{23} is applied, the particle stay in the lowest Landau level and only transverse x^1 direction is available for the free motion. This is so called "dimensional reduction" due to the magnetic field. The particle moves in the effective $1+1$ dimensions whose kinematic effect gives Coulomb's law of linear potential. This is closely parallel to the fact in metric: g_{22} and g_{22} is highly suppressed relative to g_{tt} and g_{11}.

V. SUMMARY AND DISCUSSION

In this paper, we study the UV behavior of the Wilson loop in the noncommutative gauge theory. The Wilson loop calculation in AdS/CFT is reduced to the particle dynamics in a potential defined by the D3-brane charge and NS-NS B

field. In spite of the the lack of the gauge invariance of the Wilson loops in non-commutative gauge theory, a physically meaningful aspect of Wilson loop comes out. After calculating various cases, we observed that the area law in the UV region is universal if no B_{01} is applied and it is consequence of balance of two competing tendency: the effect of F_5 flux $[H(u)]$ is to pull out the particle to the boundary of AdS, while that of B_{23} $[h(u)]$ is to pull in the particle into the horizon.

In case of the D3-brane, the effect has striking similarity with so called dimensional reduction and ''Magnetic catalysis,'' where the strong magnetic field projects the electron states to its lowest Landau level so that the charged particle has reduced degrees of freedom: it is effectively $(1+1)$-dimensional system [31,32]. If the magnetic field F_{23} is turned on, the x^2,x^3 plane is effectively confining the electron motion and the system undergoes dimensional reduction, which in turn causes chiral symmetry breaking of a massless fermion system. Apparently, the similarity between the charged particle and open string in strong magnetic field is not complete, since the string is dipole rather than a charge. If the string aligned along the x^1 direction transverse to the non-commutative plane, it does not see the dimensional reduction at all. However, the Wilson line we discussed is with zero velocity and the particle with zero velocity does not feel any magnetic field nor the dimensional reduction, either. So, the parallelism is stronger than expected. So, it would be interesting to study whether magnetic catalysis phenomena exist in the $(3+1)$-dimensional noncommutative field theory.

ACKNOWLEDGMENTS

This work was supported by Korea Research Foundation Grant (KRF-2000-015-DP0081) and by the Brain Korea 21 Project of 2001.

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